Prof. Dr. Ulrich Bunke

Universität Regensburg
WS 2022/23

Noncommutative Homotopy theory II
Exercise 1

hand in until: 24.04.2023

Exercise 1. Let G be a topological group and G^δ be G with the discrete topology. We have a canonical homomorphism $G^\delta \to G$. Show that $\mathrm{Res}^G_{G^\delta} : GC^*\mathbf{Alg}^{\mathrm{nu}} \to G^\delta C^*\mathbf{Alg}^{\mathrm{nu}}$ is the left-adjoint of a right Bousfield localization. Conclude that $GC^*\mathbf{Alg}^{\mathrm{nu}}$ is complete and cocomplete and derive formulas for limits and colimits in $GC^*\mathbf{Alg}^{\mathrm{nu}}$ in terms of limits and colimits in $G^\delta C^*\mathbf{Alg}^{\mathrm{nu}}$.

Exercise 2. Let H be a closed subgroup of G and A be in $HC^*\mathbf{Alg}^{\mathrm{nu}}$. We consider the G^δ-C^*-subalgebra

$$\{f \in C_b(G, A) \mid (\forall g \in G, \forall h \in H \mid f(gh) = \alpha_{h^{-1}} f(h))\}$$

of the G^δ-C^*-algebra $C_b(G, A)$ with the left-regular action. We let $\mathrm{Ind}^G_H(A)$ be the closure of the subalgebra of functions such that $\mathrm{pr}_{G/H}(\mathrm{supp}(f))$ is compact in G/H. Show that $\mathrm{Ind}^G_H(A) \subseteq C_b(G, A)^c$, where $(-)^c$ takes the subalgebra of continuous elements (the right adjoint in Ex. 1).

Exercise 3. Let H be a closed subgroup. Show that the restriction $\mathrm{Res}^G_H : GC^*\mathbf{Alg}^{\mathrm{nu}} \to HC^*\mathbf{Alg}^{\mathrm{nu}}$ has a right-adjoint and describe this right-adjoint explicitly.

Exercise 4. We consider the group of p-adic integers defined as the limit

$$\mathbb{Z}_p := \lim_n \mathbb{Z}/p^n\mathbb{Z}$$

in topological groups, where $\mathbb{Z}/p^n\mathbb{Z}$ has the discrete topology. Describe the normalized Haar measure μ on $\mathbb{Z}_p$. Calculate $\int_{\mathbb{Z}_p} |g|_p^s \mu(g)$ for $\mathrm{Res}(s) \geq 0$.