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Noncommutative Homotopy theory II
Exercise 2

hand in until: 24.04.2023

Exercise 1. Calculate the Haar measure and the modular function of the $ax+b$-group $\mathbb{R} \rtimes \mathbb{R}^*$.

Exercise 2. Let (A, α) be a G-C^*-algebra and consider the convolution product $*$ on $C_c(G, A)$ given by $(f * f')(g) = \int_G f(h)\alpha_h f(h^{-1}g)\mu(g)$ and the involution $*$ given by $f^*(g) := \Delta(g)^{-1}\alpha_g f(g^{-1})^*$. Show that the convolution product is associative and that the involution is really one with $(f * f')^* = f'^{,*} * f^*$.

Exercise 3. Let $\hat{G} := \mathrm{Hom}(G, U(1))$ be the character group of G with the compact open topology. Show that $\hat{G}$ acts continuously on $A \rtimes G$ such that for ξ in $\hat{G}$ we have $(\xi f)(g) := \xi(g)f(g)$. Conclude that $- \rtimes G$ refines to a functor $GC^*\mathbf{Alg}^{\mathrm{nu}} \to \hat{G}C^*\mathbf{Alg}^{\mathrm{nu}}$.

Exercise 4. Let r be in $\mathbb{R}$. Consider the C^*-algebra $C(\mathbb{R}/\mathbb{Z})$ with the action of $\mathbb{Z}$ given by $(\alpha_n f)([t]) = f([t - nr])$. Show that $C(\mathbb{R}/\mathbb{Z}) \rtimes \mathbb{Z}$ is the universal algebra generated by two unitaries U, V with $UVU^*V^* = e^{2\pi i r}$.