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Noncommutative Homotopy theory II
Exercise 3

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Exercise 1. Assume that G is a compact group. Let $\epsilon : \mathbb{C} \to K(L^2(G) \otimes \ell^2) = K_G$ be the homomorphism given by the projection onto the space of constant functions on G and the multiples of the first basis vector of ℓ^2. Show that (K_G, ϵ) is a tensor idempotent in $GC^*\mathbf{Alg}^{\mathrm{nu}}_h$.

Exercise 2. Show: If G is not compact, then K_G does not contain any finite-dimensional projection.

Exercise 3. Let H be a closed subgroup of G. Let A be in $HC^*\mathbf{Alg}^{\mathrm{nu}}$ and B be in $GC^*\mathbf{Alg}^{\mathrm{nu}}$. Show that $\mathrm{Ind}_H^G(A \otimes \mathrm{Res}_H^G(B)) \cong \mathrm{Ind}_H^G(A) \otimes B$ in $GC^*\mathbf{Alg}^{\mathrm{nu}}$.

Exercise 4. Let G be a locally compact abelian group. Show that the left-regular representation of G on $L^2(G)$ together with the action of $\hat{G}$ by multiplication operators $(\chi, f) \mapsto (g \mapsto \chi(g)f(g))$ generate $K(L^2(G))$.