Prof. Dr. Ulrich Bunke
Universität Regensburg
WS 2022/23

Noncommutative Homotopy theory II
Exercise 3

hand in until: 15.05.2023

Exercise 1. Let G be a locally compact group and (B, β) be a G-C^* algebra. Let v in $M(B)$ be an isometry (i.e. $v^*v = 1$). Then $\sigma_g := \beta_g(v^*)v$ defines a cocycle $G \to U(M(B))$.

Exercise 2. We let $K := K(\ell^2)$ and $e_{i,j}$ be the usual 1-dimensional generators. Let (A, α) be a G-C^*-algebra and $\tilde{\alpha}$ be a G-action on $A \otimes K$ such that $A \to A \otimes K$ given by $a \mapsto a \otimes e_{11}$ is equivariant. Show that $\sigma_g := \sum_{i \in \mathbb{N}} \tilde{\alpha}_g(1 \otimes e_{i,1})(1 \otimes e_{1,i})$ defines a cocycle $G \to U(M(A \otimes K))$ extending the identity $(A \otimes K, \alpha \otimes \mathrm{id}_K) \to (A \otimes K, \tilde{\alpha})$ to a weakly equivariant map.

Exercise 3. Let (A, α) and (B, β) be G-C^*-algebras. Let $(f, \sigma) : A \to B$ be a weakly equivariant map. Check that the induced map $k \mapsto \sigma_g f(k(g, g'))\sigma^*_{g'}$ is an equivariant map $A \otimes K(L^2(G)) \to B \otimes K(L^2(G))$. Here we use A-valued integral kernels $k(g, g')$ in order to present elements of $A \otimes K(L^2(G))$.

Exercise 4. Show that two projections in K are Murray-von Neumann equivalent if and only if they have the same dimension.