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Noncommutative Homotopy theory II
Exercise 5

hand in until: 05.06.2023

Exercise 1. Let H be a closed subgroup of a locally compact group G and G/H be compact. Since Coind_H^G is the right-adjoint of the symmetric monoidal functor Res_H^G the functor Coind_H^G is lax symmetric monoidal. It therefore preserves algebras. Therefore $\mathrm{Coind}_H^G(\mathbb{C})$ is an algebra. Describe the product explicitly.

Exercise 2. Consider the cyclic group C_3. Determine all projections in $C_r^*(C_3)$ explicitly.

Exercise 3. Let G be discrete and consider the algebra $C^*_{\max}(G) := \mathbb{C} \rtimes G$. Show that the linear map $f \mapsto f(e)$ from $C_c(G)$ to $\mathbb{C}$ extends to a trace $\tau : C^*_{\max}(G) \to \mathbb{C}$.

Exercise 4. Let $\tau : A \to \mathbb{C}$ be a trace on a C^*-algebra A. Show that the map $\mathrm{Hom}_{C^*\mathbf{Alg}^{\mathrm{nu}}}(\mathbb{C}, A) \to \mathbb{C}$, $f \mapsto \tau(f(1))$ canonically extends to a semigroup map $\tau : \pi_0\mathrm{Map}_{L_K C^*\mathbf{Alg}^{\mathrm{nu}}_h}(\mathbb{C}, A) \to \mathbb{C}$.