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Noncommutative Homotopy theory II
Exercise 6

hand in until: 26.06.2023

Exercise 1. Show that there exists an internal Hom-functor (actually two the two versions tensor products)

$$\underline{\mathbf{KK}}^G : \mathbf{KK}^{G,\mathrm{op}} \otimes \mathbf{KK}^G \to \mathbf{KK}^G$$

characterized by

$$\mathbf{KK}^G(A, \underline{\mathbf{KK}}^G(B,C)) \simeq \mathbf{KK}^G(A \otimes B, C)\ , \quad A, B, C \in \mathbf{KK}$$

Show further, that for a closed subgroup H

$$\underline{\mathbf{KK}}^H(\mathrm{Res}^G_H(B), \mathrm{Res}^G_H(C)) \simeq \mathrm{Res}^G_H(\underline{\mathbf{KK}}^G(B,C))\ .$$

Exercise 2. Let G be a finite group and H be a subgroup. Show that $\mathrm{kk}^G(C(G/H))$ is selfdual: There is a binatural equivalence

$$\mathbf{KK}^G(A \otimes C(G/H), B) \simeq \mathbf{KK}^G(A, C(G/H) \otimes B)\ , \quad A, B \in \mathbf{KK}^G$$

and

$$\mathrm{kk}^G(C(G/H)) \simeq \underline{\mathbf{KK}}^G(C(G/H), \mathbb{C})\ .$$

Exercise 3. Consider the functor

$$P^2 : GC^*\mathbf{Alg}^{\mathrm{nu}} \to GC^*\mathbf{Alg}^{\mathrm{nu}}\ , \quad A \mapsto A \otimes A\ .$$

Show that this functor has an essentially unqiue factorization through a filtered colimit preserving functor $\mathbf{KK}^G \to \mathbf{KK}^G$

$$\begin{array}{ccc} GC^*\mathbf{Alg}^{\mathrm{nu}} & \xrightarrow{P^2} & GC^*\mathbf{Alg}^{\mathrm{nu}} \\ \downarrow{\scriptstyle \mathrm{kk}^G} & & \downarrow{\scriptstyle \mathrm{kk}^G} \\ \mathbf{KK}^G & \dashrightarrow & \mathbf{KK}^G \end{array}\ .$$

Exercise 4. Determine the representation ring $R(S_3)$, where S_3 is the symmetric group. Describe an additive basis and the product table.