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Noncommutative Homotopy theory II
Exercise 7

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Exercise 1. Let X be a locally compact Hausdorff space with a proper and cocompact action of a locally compact group G. Show that there exists a non-negative function χ in $C_c(X)$ such that $\int_G \chi(g^{-1}x)^2\mu(x) = 1$ for all x in X. Show further that the space of such functions is contractible.

Exercise 2. Let G be a discrete group, H be a subgroup of G such that G/H is finite, and let B be in $GC^*\mathbf{Alg}^{\mathrm{nu}}$. Show that $\underline{\mathbf{KK}}^G(C_0(G/H), B) \rtimes G \simeq \mathrm{Res}^G_H(B) \rtimes H$.

Exercise 3. Let p be a prime. Calculate the completion of $R(C_p)$ at the dimension ideal explicitly.

Exercise 4. Let C_p act on S^2 be rotation along the z-axis. Calculate $\mathrm{K}^{C_p}(S^1)$ after localization at the dimension ideal in $R(C_p)$ using the localization theorem and directly from a CW-decomposition. Compare the results.