Prof. Dr. Ulrich Bunke
Universität Regensburg
WS 2022/23

Noncommutative Homotopy theory II

Exercise 9

hand in until: 17.07.2023

Exercise 1. Let G be a discrete group and $E : G\mathrm{Orb} \to \mathbf{Sp}$ be a functor. Let $\mathcal{F}$ be a family of subgroups and $i : G_{\mathcal{F}}\mathrm{Orb} \to G\mathrm{Orb}$ be the inclusion. Show that the Davis-Lück assembly map $i_! i^* E(*) \to E(*)$ is equivalent to the map $E(E_{\mathcal{F}}G) \to E(*)$ induced by the projection to the point.

Exercise 2. Let G be a discrete group and $E : G\mathrm{Orb} \to \mathbf{Sp}$ be a functor. Let $\mathcal{F}$ be a family of subgroups and $i : G_{\mathcal{F}}\mathrm{Orb} \to G\mathrm{Orb}$ be the inclusion. Let H be a subgroup of G and $j : H\mathrm{Orb} \to G\mathrm{Orb}$ be the induction. Show that $i_! i^* E(G/H) \to E(G/H)$ is equivalent to the Davis-Lück assembly map for j^*E and the family $\mathcal{F} \cap H$.

Exercise 3. Let G be a discrete group, $\mathcal{F}$ be a family of subgroups, and $E, E' : G\mathrm{Orb} \to \mathbf{Sp}$ be two functors with an equivalence $E_{|G_{\mathcal{F}}\mathrm{Orb}} \simeq E'_{|G_{\mathcal{F}}\mathrm{Orb}}$. Show that if X is a G-topological space with stabilizers in $\mathcal{F}$, then we get an induced equivalence $E(X) \simeq E'(X)$.

Exercise 4. Calculate the homotopy groups of the domain $R\mathrm{KK}^{\mathbb{Z}}(E_{\mathcal{F}in}\mathbb{Z}, \mathbb{C}, \mathbb{C})$ of the Kasparov assembly map $\mu^{Kasp}_{\mathbb{Z},\mathbb{C},\mathbb{C}}$.

Exercise 5. * Let B be a unital C^*-algebra and $\mathrm{Mod}(B)^{fp}$ be the C^*-category of finitely generated projective Hilbert-B-modules and compact operators. We can consider B as an object of $\mathrm{Mod}(B)^{fp}$. Show that the inclusion $B \to \mathrm{Mod}(B)^{fp}$ induces an equivalence $K(B) \to K(A(\mathrm{Mod}(B)^{fp}))$.

Hint: Write $A(\mathrm{Mod}(B)^{fp})$ *as a filtered colimit of* $A(-)$ *applied to subcategories with finitely many objects including* B*. If* $\mathbf{C}$ *is such a subcategory construct a Morita bimodule exhibiting a Morita equivalence between* B *and* $A(\mathbf{C})$*. Finally use that* K *commutes with filtered colimits and is Morita equivariant.*